2017 Workshop on Nonlinear PDEs in Applied Mathematics

Izmir Institute of Technology, Izmir, TURKEY

August 8-10, 2017
Support

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Long Talks: 50 min (45+5)
Short Talks: 25 min (20+5)
Abstracts

* refers to the person who will be delivering the talk.
Boundary effect in the zero viscosity limit and energy dissipation

Claude Bardos*, László Szekelyhidi, E. Titi
University of Paris 7- Denis Didero

This talk is based on the two following observations. First the notion of week convergence is the deterministic avatar of the notion of average in the statistical theory of turbulence. Second in most cases (even for generation of homogeneous and isotropic turbulence), the basic effects are due to the boundary. It turns out that it is a theorem of T. Kato of 1984 that allows through diverse generalizations (object of this presentation) to make a natural connection between these two observations; moreover what is striking is that the hypothesis of this theorem are in full agreement with observations coming from engineering science or fluid mechanic. Eventually the fact that (even in domain with boundary) weak solutions of the Euler equations are of constant energy as soon as they belong to an Holder space $C^{0,\alpha}(\Omega)$ with $\alpha > \frac{1}{3}$ (the so called Onsager’s conjecture) leads to a comparison between the Kolmogorov $1/3$ law and the Kato hypothesis for the absence of energy dissipation.
The Resonant Soliton Hierarchies. From RNLS and KP-II to Relativistic and q-Dispersive Resonant Solitons

Oktay Pashaev*
Izmir Institute of Technology

In this talk I review integrable resonant soliton hierarchies and describe some new results and applications. I will start from $SL(2,R)$ AKNS hierarchy, the second flow of which is related with resonant $NLS$ ($RNLS$) equation in $1+1$ dimensions. This equation appears in cold plasma physics and includes de Broglie-Bohm quantum potential. Soliton interactions for $RNLS$ equation show the resonance phenomena and in non-Madelung fluid representation it is related with the Broer-Kaup resonant system. The Malomed-Stenflo generalized NLS as shown, in special domain of parameters admits soliton resonances and can be transformed to RNLS. Combining the second and the third flows of the hierarchy we construct resonant line solitons of $KP-II$ equation in $2+1$ dimensions. As a next natural generalization we discuss the resonant DS equation, $2+1$ dimensional nonlocal $RNLS$ and corresponding $2+1$ dimensional BroerKaup type system. By the recursion operator of AKNS hierarchy, the relativistic and q-dispersive resonant soliton equations are derived on this hierarchy. Similar approach is applied to the Kaup-Newell hierarchy and resonant $DNLS$ equation with chiral soliton interactions. The second and the third flows for this hierarchy describe resonant solitons of MKP-II. By the recursion operator the relativistic resonant $DNLS$ equation is constructed and q-dispersive resonant equations are discussed.

This work is supported by TUBITAK Grant 116F206.

Bibliography


In this research we study a Cauchy problem for the nonlinear damped wave equations for a general positive operator with discrete spectrum. We derive the exponential in time decay of solutions to the linear problem with decay rate depending on the interplay between the bottom of the operator’s spectrum and the mass term. Consequently, we prove global in time well-posedness results for semilinear and for more general nonlinear equations with small data. Examples are given for nonlinear damped wave equations for the harmonic oscillator, for the twisted Laplacian (Landau Hamiltonian), and for the Laplacians on compact manifolds.
We present a class of phase-field models based on the double-obstacle Ginzburg-Landau functional. They involve vector-valued potentials with values constrained on convex sets beyond the classic Gibbs complex, and are therefore suitable for the modeling of multi-phase problems with various complicated domain/junction arrangements. We discuss their efficient variational discretization, based on De Giorgi’s minimizing movements, and certain interesting geometrical insights concerning their behavior in the sharp limit.

Bibliography


August 8th
13:00-13:50

TBA
Nader Masmoudi*
New York University

To be announced...
1. Introduction. The Quantum Fluid Dynamics (QFD) representation of quantum mechanics was partly motivated by questions of the completeness of the quantum theory, while admitting to its internal consistency. The QFD representation has its foundations in Madelung’s work (1926) in the early times and followed by Bohm’s interpretation of quantum mechanics (1950’s) with the goal to find classically identifiable dynamical variables at the sub-particle level [1]. The approach leads to conservation laws for ”mass”, ”momentum” and ”energy” similar to those in hydrodynamics for a compressible, non dissipative fluid.

The QFD equations are a set of nonlinear partial differential equations. In this sense, they may be seen as a step in the negative direction as compared to the linear Schrödinger equation. However, in this scheme, the oscillatory real and imaginary components of the complex wave function are replaced by the monotonous the amplitude and phase. This has been exploited as a significant advantage in computational quantum mechanics [1, 2].

In this study, the nonlinear field equations are suggested as the natural framework for studying fundamentally nonlinear phenomena such as solitons and chaos, in a multi-dimensional context. This expose extends the QFD formalism to the case of a non-isolated medium. The interaction of this analogous fluid medium with the environment is postulated to lead to a dissipative piece in the acceleration and is introduced phenomenologically as in the classical Navier - Stokes equation.

The Schrödinger equation being linear and having basic conservation laws for momentum and energy does not show possibilities for ”turbulence”. As for the Navier - Stokes equation, this formulation ”may” contain a necessary ingredient of chaotic behavior for quantum mechanics. This modified form allows for vorticities that may break into a cascade, eventually leading to ”turbulence”, as in the classical Navier - Stokes equation.

2. From Schrödinger’s to Quantum Fluid Dynamics (QFD) Equation. The starting point is the Schrödinger equation for the complex valued wave function \( \psi \), nondimensionalized by reducing the Planck’s constant \( \hbar \) of the conventional quantum mechanics and the mass \( m \) of the particle to unity is:

\[
\frac{i \hbar}{\partial t} \psi = -\frac{1}{2} \nabla^2 \psi + V \psi
\]

\( V = V(x, t) \) is the externally applied potential energy that guides the dynamics and \( i = \sqrt{-1} \) is the usual imaginary unit.

The complex wave function \( \psi \) is represented by its amplitude \( A \) and phase \( S \) as:

\[
\psi(x, t) = A(x, t) \ e^{i[S(x, t) - Et]}
\]
Above by reference to quantum mechanics, E is singled out to be identified as at the total energy in the derivations below.

The substitution of the representation of the wave function in the polar form in the Schrödinger equation, the separation of its respectively imaginary and real parts and the definition of the density ρ, velocity v, the externally applied force g = −∇V and the "Quantum potential" or "pressure" yield the hydrodynamics representation of quantum mechanics. The equations are respectively "Mass conservation" and "Momentum conservation" equations:

\[ \rho = A^2 \quad v = \frac{\nabla S}{m} \quad V_q = -\frac{1}{2} \frac{\nabla^2 A}{A} \quad g = -\nabla V \]

\[ \frac{\partial \rho}{\partial t} + \nabla \cdot (\rho v) = 0 \]

\[ \frac{\partial v}{\partial t} + v \cdot \nabla v = -\nabla V_q + g \]

These are called alternatively as "Madelung’s quantum hydrodynamics equations", "Bohmian representation of quantum mechanics", "Quantum fluid dynamics (QFD) equations"...

The energy equation is also derived as an alternative to the momentum equation:

\[ \frac{\partial S}{\partial t} + \frac{1}{2} v \cdot v + V + V_q = E \]

S is interpreted as an action whose time rate S, is an energy; \( \frac{1}{2} mv \cdot v + V \) form the classical energy with its kinetic and potential energy components; and \( V_q \) as a non-local "quantum potential" as the Laplacian connects neighboring regions.

The linear and nonlinear parts of the momentum equation are the local and convective acceleration terms of classical fluid continua as in the Navier - Stokes equations. The acceleration in the Navier - Stokes equation contains also a diffusive component represented by the phenomenological term \( -\mu \nabla^2 v \). \( \mu \) is called as "viscosity". Adding this term to the momentum equation above leads to:

\[ \frac{\partial v}{\partial t} + v \cdot \nabla v = -\nabla (V + V_q) - \mu \nabla^2 v \]

3. Further use of QFD. The same decomposition of the wave function can also be applied to the Nonlinear Schrödinger equation that leads to the same mass conservation equation and slightly modified momentum and energy equations:

\[ i \frac{\partial \psi}{\partial t} = -\frac{1}{2} \nabla^2 \psi - k^2 |\psi|^2 \psi \quad \rightarrow \quad \frac{\partial v}{\partial t} + v \cdot \nabla v = -\nabla (V_q + k^2 \rho) \]

\[ \frac{\partial S}{\partial t} + \left( \frac{1}{2} |v|^2 + V_p + k^2 \rho \right) = E \]

QFD allows also the representation Gross - Pitaevskii equation for Bose-Einstein condensation.
References


[2] (Sample of the Work by AA’s contributions over the years)
Remark on the eigenvalues of the $m$-Laplacian in a punctured domain

Gulzat Nalzhupbayeva*
Al-Farabi Kazakh National University

In this research we extend results on regularized trace formulae which were established by Kanguzhin and his co-authors for the Laplace and $m$-Laplace operators in a punctured domain with the fixed iterating order $m \in \mathbb{N}$. By using techniques of Sadovnichii and Lyubishkin, Kanguzhin and his co-authors described regularized trace formulae in the spatial dimension $d = 2$. In this remark one is to be claimed that the formulae are also valid in the higher spatial dimensions, namely, $2 \leq d \leq 2m$. Also, we give the further discussions on a development of the analysis associated with the operators in punctured domains. This can be done by using so called ‘nonharmonic’ analysis.
Data assimilation refers to the process of completing, or enhancing the resolution of the initial condition. The idea behind continuous data assimilation is to assimilate observational data into a model as it is being integrated in time so that an approximate solution converges to the true solution [1, 2]. In this talk we introduce an algorithm for continuous data assimilation for the two-dimensional g-Bénard convection problem by following Farhat, Jolly and Titi [3].

Bibliography


Global stabilization of solutions to the complex Ginzburg-Landau equation by finite parameter controllers

Jamila Kalantarova*, Türker Özsarı
Izmir Institute of Technology

This talk concerns the initial boundary value problem for the complex Ginzburg-Landau equation:

\[ u_t - (\lambda + i\alpha)\Delta u + (\kappa + i\beta)|u|^p u - \gamma u = \Pi(u), \quad x \in \Omega, \; t > 0, \]

where \( u \) denotes the complex oscillation amplitude, \( \beta \in \mathbb{R} \) and \( \alpha \in \mathbb{R} \) are the (nonlinear) frequency and (linear) dispersion parameters, respectively. The constants \( \lambda \) and \( \kappa \) are assumed to be strictly positive. \( \Omega \subset \mathbb{R}^n \) is a bounded domain and \( p > 0 \) is the source-power index. The operator \( \Pi \) at the right hand side denotes a finite parameter feedback controller, which employ finitely many volume elements, Fourier modes or nodal observables. We prove the global exponential stabilization of this problem. Moreover, steering solutions of the Ginzburg-Landau equation to a desired solution of the non-controlled system employing finite parameter controllers is established.

Both researchers in this work were supported by TÜBİTAK 3501 Career Grant 115F055
A new modified schemes for shallow water linear equations and Burgers nonlinear equation

Abdelhamid Laouar*
Badji Mokhtar University of Annaba

This work concerns the study of the linear equations of the shallow water and the nonlinear equation of Burgers for a perfect fluid and a weakly viscous fluid. For the equations of shallow water, the phenomenon is modeled by a system of improved equations of Boussinesq. We add to them the numerical dispersion due to truncation errors (discretization errors). For the solution, we adopt the finite difference method combined with the implicit scheme of the alternate direction. In the same way, we proceed for the nonlinear equation of Bourgers. Then, we study the stability and the convergence of the solution and make a comparison with some existing results. Finally for illustration, we present numerical simulations.

Keywords: Boussinesq equations, Bourgers equation, Modified schemes, Stability and convergence.
Existence of Minimal and Maximal solutions for Quasilinear Elliptic Equation with Nonlocal Boundary Conditions on Time-Scales

Mohammed Derhab, Mohammed Nehari*
E.S.M.Tlemcen

Abstract. The purpose of this work is the construction of minimal and maximal solutions for a class of second order quasilinear elliptic equation subject to nonlocal boundary conditions. More specifically, we consider the following nonlinear boundary value problem

\[
\begin{cases}
-\left(\varphi_p (u^{\Delta})\right)^{\Delta} = f(x, u), (a, b)_T, \\
u (a) - a_0 u^{\Delta} (a) = g_0 (u), \\
u (\sigma (b)) + a_1 u^{\Delta} (\sigma (b)) = g_1 (u),
\end{cases}
\]

where \( p > 1 \), \( \varphi_p (y) = |y|^{p-2} y \), \( \left(\varphi_p (u^{\Delta})\right)^{\Delta} \) is the one-dimensional \( p \)–Laplacian, \( f : [a, b]_T \times \mathbb{R} \to \mathbb{R} \) is a rd-continuous function, \( g_i : C_{rd} ([a, b]_T) \times C_{rd} ([a, b]_T) \to \mathbb{R} \) \( (i = 0 \text{ and } 1) \) are rd-continuous and \( a_0 \) and \( a_1 \) are a positive real numbers.

Key words: Quasilinear elliptic equation; Time-Scale, Nonlocal boundary conditions; upper and lower solutions; monotone iterative technique

AMS Classification: 34B10, 34B15
Description of a New Generalized Quasi Two-Phase Mass Flow Model

August 8th
17:05-17:30

Puskar R. Pokhrel*
Kathmandu University

Employing the full dimensional two-phase (mixture of solid particles and viscous fluid) mass flow model equations [1], we have generated a novel, generalized quasi two-phase, full two-dimensional model for bulk mixture flow down a channel. The emerging model is written as a well-structured system of three highly non-linear partial differential equations in conservative form representing the mass and the momentum balances. The new mechanical and dynamical concepts of generalized bulk and shear viscosities, pressures, and velocities for the mixture characterize the model. Some new reduced models are also obtained by considering different aspects of dynamics and modeling. The advantage of the model lies mainly in providing a possibility for simulating the mixture velocities and pressures much faster than the two-phase mass flow model. Furthermore, the introduction of the velocity and pressure drifts factors makes it possible to reconstruct the two-phase mass flow so as to capture its basic dynamics.

Keywords: Quasi-two-phase flow, generalized bulk and shear viscosities, Generalized mixture velocities and pressure, Velocity and pressure drift factors

Bibliography

Global Well-posedness of an Inviscid Three-dimensional Pseudo-Hasegawa-Mima-Charney-Obukhov Model

Chongsheng Cao, Aseel Farhat, Edriss S. Titi*
The Weizmann Institute of Science and Texas A&M University

The 3D inviscid Hasegawa-Mima model is one of the fundamental models that describe plasma turbulence. The same model is known as the Charney-Obukhov model for stratified ocean dynamics, and also appears in literature as a simplified reduced Rayleigh-Bénard convection model. The mathematical analysis of the Hasegawa-Mima and of the Charney-Obukhov equations is challenging due to the their resemblance with the Euler equations. In this talk, we introduce and show the global regularity of a model which is inspired by the inviscid Hasegawa-Mima and Charney-Obukhov models, named a pseudo-Hasegawa-Mima model. The introduced model is easier to investigate analytically than the original inviscid Hasegawa-Mima model, as it has a nicer mathematical structure. To establish our global regularity result we implement a new logarithmic inequality, generalizing the Brezis-Gallouet-Berzis-Wainger inequalities.
Comparison of solutions of some families of wave equations

Husnu Ata Erbay, Saadet Erbay, Albert Erkip*
Sabancı University

We will present a series of recent results concerning the comparison of solutions of two different wave-type equations in asymptotic regimes determined by two parameters characterising long waves and small amplitude. There are many works in literature on similar comparisons within the scope of fluid dynamics. These comparisons are made between a model equation, typically the Camassa-Holm (CH) equation, and a parent equation, namely the Euler equation, the Boussinesq system, or similar. In our work, we have considered the same question within the scope of nonlocal elasticity. In that respect we first show that the CH equation can be formally derived from the improved Boussinesq (IB) equation. We then give a rigorous justification of this, namely prove that solutions of the CH equation are well approximated by appropriate solutions of the IB equation. Conversely we show that solutions of the IB equation can be written as a sum of right and left going solutions of the CH equations up to a small error. These results can be generalised to nonlocal wave equations which have a similar dispersion relation as the IB equation in the long wave limit. Finally we consider the comparison of nonlocal equations and give estimates between solutions of two nonlocal wave equations.

Bibliography


The main aim of this talk is to describe the stability and dynamic transitions of a basic shear flow, associated with geophysical baroclinic instability, for the three dimensional (3D) continuously stratified rotating Boussinesq model. The model admits a steady state solution characterizing a shearing motion which, due to the Coriolis forces, must be balanced by a pressure gradient in response to spatially varying density field. We first establish thresholds for the energy stability and linear stability of this basic solution via energy method. Next by numerically computing the center manifold using a spectral method, we establish the reduced equations of the system at the onset of transition and prove the existence of various transition types describing transitions to multiple steady states as well as to spatio temporal oscillations. This is joint work with Shouhong Wang.
An inertial manifold (IM) is one of the key objects in the modern theory of dissipative systems generated by partial differential equations (PDEs) since it allows us to describe limit dynamics of the considered system by the reduced finite-dimensional system of ordinary differential equations (ODEs). It is well known that the existence of an IM is guaranteed when the so called spectral gap conditions are satisfied, whereas their violation leads to the possibility of an infinite dimensional limit dynamics, at least on the level of an abstract parabolic equation. However, these conditions restrict greatly the class of possible applications and are usually satisfied in the case of one spatial dimension only.

The main aim of this talk is a comprehensive study of reaction-diffusion-advection (RDA) equations in 1D with different boundary conditions (BC). Even in 1D case the spectral gap condition does not satisfied for RDA equations and therefore question on existence/non-existence of an IM for it requires accurate analysis. In the case of Dirichlet or Neumann boundary conditions we will show the existence of an IM using a specially designed non-local in space diffeomorphism which transforms the equations to the new ones for which the spectral gap conditions are satisfied. In contrast to this, in the case of periodic boundary conditions, we will construct a natural example of a RDA system which does not possess an IM. It is worth mentioning that scalar RDA equation even with periodic BC possesses IM, and this may be shown by using the modification of the transform proposed for Dirichlet BC.
Hyperbolic Relaxation of the 2D Navier-Stokes Equations in a Bounded Domain

Sergey Zelik*
University of Surrey

A hyperbolic relaxation of the classical Navier-Stokes problem in 2D bounded domain with Dirichlet boundary conditions is considered. It is proved that this relaxed problem possesses a global strong solution if the relaxation parameter is small and the appropriate norm of the initial data is not very large. Moreover, the dissipativity of such solutions is established and the singular limit as the relaxation parameter tends to zero is studied.
A topological theory of local and global Lyapunov exponents for abstract dynamical systems was introduced by Eden, Foias and Temam [2] in the late 1980’s. Although the theory was developed in order to obtain better dimension upper estimates for the global attractors for dissipative and/or damped PDE’s, it had more conclusive results for low dimensional dynamical systems such as the Lorenz system of ODE’s. Three questions were posed in a conference in Luminy in 1989 seems to have led to some interesting research [1]. A series of results by Leonov in 1990’s led to the positive resolution of the third question about the local Lyapunov dimension of the Lorenz attractor [3].

There has been a renewed interest in the field by Leonov, Kuznetsov and co. (see e.g. [4, 5]) In my talk, I will take a nostalgic journey from the beginning of this theory to some of the more recent results. The talk will not be technical.

Bibliography


Determination of the Wall Function for Navier-Stokes Solutions on Cartesian Grids

Emre Kara*
University of Gaziantep

Cartesian grids have beneficiary effects in the solution of partial differential equations (PDEs) resulting from their characteristic form of discretized finite-volume representation, i.e. nonlinear Navier Stokes equations involving irregular and multiple boundaries.

A new flow solver is generated in Visual Studio by using object-oriented FORTRAN programming and is called GeULER-NaTURe (cartesian Grid generator with EULER Navier Stokes TURbulent flow solvEr) [1]. Viscous terms of Navier-Stokes equations and Spalart-Allmaras [2] turbulence model are implemented into solver.

Near the solid boundary, generally, wall function is established by universal law-of-the-wall coordinate, \( y^+ \), presented by Spalding [3].

GeULER-NaTURe flow solver [1] is dealing with external turbulent flows with high Reynolds numbers around embedded boundaries, so that the wall-bounded flow approach should be followed to limit the resolution of the fine grids in the turbulent boundary layer. In this region, the wall function is employed for the coarse grid utilization at \( 30 \leq y^+ \leq 150 \) instead of fine grids at \( y^+ < 5 \) thus eliminates stiffness problem owing to redundant refinement. The analytic solution suggested by Berger et al. [4] in place of Spalding formula is selected as the wall function predictor, since the proposed formula is an explicit function of velocity in contrast to Spalding formula. Turbulent viscosity required for SA relations is adapted from Frink’s study [5].

Bibliography


An Active Control of a Double-Beam Complex System

Melih Çınar*
Yıldız Technical University

This paper deals with an active optimal control of free transverse vibrations of an elastically bonded continuous beam-string system. The aim of the paper is to minimize the physical energy through displacement and the velocity of the system. For this purpose, the actuators are placed in the domain of the system. A performance index functional which comprises of modified energy functional of the system and the expenditure of the actuators is also introduced. To minimize the performance index functional calculus of variation is used and the necessary conditions for the optimality are derived in the form of Fredholm integral equations. A numerical example is illustrated to show the applicability of the proposed technique. [1] [2] [3] [4] [5].

Bibliography


Well-posedness of the fourth-order Schrödinger equation on the half-line in fractional Sobolev spaces

Türker Özsarı, Nermin Yolcu*
Izmir Institute of Technology

In this talk, we consider the initial boundary value problem for the fourth order Schrödinger equation on the right half-line in $L^2$—based fractional Sobolev spaces:

\[ iq_t + \Delta^2 q = 0, \quad (x, t) \in \mathbb{R}_+ \times (0, T), \]
\[ q(x, 0) = q_0 \in H^s(\mathbb{R}_+), \]
\[ q(0, t) = g_0(t) \in H^{2s+3}_t (0, T), \]
\[ q_x(0, t) = g_1(t) \in H^{2s+1}_t (0, T). \]

At the beginning, we briefly introduce the Fokas method (also known "Unified Transform Method (UTM)"), which is a novel approach for obtaining representation formulas for the solutions of the initial-boundary value problems posed on the half-line. The representation formula obtained via UTM is then analyzed by using the Fourier theory to deduce sharp estimates on the space and time traces of the solution from which the well-posedness follows.
Analytical solution of the Volterra-Fredholm singular integro-differential

August 9th
16:15-16:40

Fernane Khairededdine*
University of 8 May 1945 Guelma

In this paper, we apply Taylor’s approximation and then transform the given nth-order weakly singular linear Volterra and Fredholm integro-differential equations with into an ordinary linear differential equation. Some different examples are considered the results of these examples indicated that the procedure of transformation method is simple and effective, and could provide an accurate approximate solution or exact solution.

We consider the following nth-order linear-weakly singular Fredholm and Volterra integro-differential equations at the form :

\[ y^{(n)}(t) + \sum_{m=0}^{n-1} \beta_m(t) y^{(m)}(t) = f(t) + \int_a^b \frac{y^{(p)}(s)}{|t-s|^\alpha} ds, \quad a \leq t \leq b, \quad 0 < \alpha < 1 \quad (2) \]

with initial conditions

\[ y(a) = \mu_0, y'(a) = \mu_1, \ldots, y^{(n-1)}(a) = \mu_{n-1} \quad (3) \]

\[ y^{(n)}(t) + \sum_{m=0}^{n-1} \beta_m(t) y^{(m)}(t) = f(t) + \int_a^t \frac{y^{(p)}(s)}{(t-s)^\alpha} ds, \quad a \leq t \leq b, \quad 0 < \alpha < 1 \quad (4) \]

with initial conditions

\[ y(a) = \mu_0, y'(a) = \mu_1, \ldots, y^{(n-1)}(a) = \mu_{n-1} \quad (5) \]

where \( \mu_i (i = 1, \ldots, n-1) \) are real constants, \( n \); \( p \) are positive integers and \( 0 \leq p \leq n \); \( f(t); \beta_k(t) (k = 0, 1, \ldots, n-1) \) are given functions, and \( y(t) \) is the solution to be determined.

The objectives of this work is to develop a new approach to resolve the high-order linear weakly-singular Fredholm and Volterra integro-differential equations in one dimensional space. The proposed method is a direct method in which we remove the singularity by using Taylor’s approximation and rewrite the weakly-singular integro-differential Fredholm and Volterra equation as either a linear differential equation that can be solved using the analytical method or the known numerical methods.

Bibliography


Non-Homogeneous Continuous and Discrete Gradient Systems: The Quasi-convex case

Vahid Mohebbi*
Roi de Janeiro IMPA

In this paper, first we study the weak and strong convergence of solutions to the following first order nonhomogeneous gradient system

$$-x'(t) = \nabla \phi(x(t)) + f(t), \quad a.e \ on \ (0, \infty)$$

$$x(0) = x_0 \in H$$

to a critical point of $\phi$ where $\phi$ is $C^1$ quasi-convex function on a real Hilbert space $H$ with $\text{Arg min} \phi \neq \emptyset$. These results extend the results of [?] to nonhomogeneous case. Then the discrete version of the above system by backward Euler discretization has been studied. Beside of the proof of the existence of the sequence given by the discrete system, some results on the weak and strong convergence to the critical point of $\phi$ are also proved. These results when $\phi$ pseudo-convex (therefore the critical points are the same minimum points) may be applied in optimization for approximation of a minimum point of $\phi$.

Key words: Gradient system, quasi-convex, backward Euler discretization, weak convergence, strong convergence.

Mathematics Subject Classification: 34D20; 37C75; 93D09
Existence and Continuation Theorems of Caputo Type Fractional Differential Equations

Shahzad Sarwar*
Shanghai University

In this talk, we consider the general fractional differential equation (FDE) with Caputo derivative and study the existence and continuation of solution. Firstly, we construct a new local existence theorem. Then we extend the continuation theorems for ODEs to those of general FDEs. Besides, several global existence results for FDE are constructed.
We consider a mixed problem for the Schrodinger equation on a finite dimensional Riemannian manifold with magnetic and electric potential coefficients and non-homogeneous Dirichlet boundary term. The goal is the nonlinear problem of the recovery of the electric potential coefficient by means of only one boundary measurement on an explicitly identified portion of the boundary. We obtain global uniqueness of the recovery and Lipschitz stability of the recovery on an arbitrarily short time-interval exclusively in terms of the data of the problem. The norms involved are optimal. The key ingredients are:

(i) sharp Carleman-type estimates for the Schrodinger equation at the H1-level, on a Riemannian manifold and consequent continuous observability estimates at the H1-level (Triggiani-Xu, AMS,2007);

(ii) related continuous observability estimates at the L2-level (Lasiecka-Triggiani-Zhang; and Triggiani 2004)

(iii) optimal interior and boundary regularity of the direct mixed problem (Lasiecka-Triggiani, 1991) plus a new boundary trace result in the present contribution by Shouhong Wang.
Error analysis of a semi-discrete method for nonlocal nonlinear wave equations

Hüsnü Ata Erbay*, Saadet Erbay, Albert Erkip
Ozyegin University

In this study we present a semi-discrete numerical method for solving the initial-value problem of the nonlocal nonlinear wave equation [1]

\[ u_{tt} = (\beta \ast f(u))_{xx}. \]

Here the symbol \( \ast \) is used to denote the convolution operation in the spatial domain

\[ (\beta \ast v)(x) = \int_{\mathbb{R}} \beta(x - y)v(y)dy \]

and the kernel \( \beta \) is an even function with \( \int_{\mathbb{R}} \beta(x)dx = 1 \). The numerical method is based on a uniform spatial discretization of the convolution integral. We provide a rigorous convergence analysis and demonstrate the second-order accuracy in space. The main ingredient in the convergence analysis is the uniform estimate of the second-order derivative of the kernel function. To confirm our theoretical findings, we apply the scheme to two different situations: the propagation of a single-solitary wave and the finite-time blow-up of solutions.

Bibliography

Stability estimates for some inverse problems for ultrahyperbolic Schrödinger equations

Fikret Gölsenleyen*, Özlem Kaytmaz
Bülent Ecevit University

In this work, we obtain a global Carleman estimate for an ultrahyperbolic Schrödinger equation which arises in several applications [2, 7]. Based on the idea by Imanuvilov and Yamamoto [3], we prove Hölder stability for the inverse problem of determining a coefficient or a source term in the ultrahyperbolic Schrödinger equation by some lateral boundary data. To our best knowledge, there is no result available in the mathematical literature related to the inverse problems for these equations. As for the classical Schrödinger equation, we refer to Baudouin and Puel [1], Lasiecka et al. [4], Mercado et al. [5], and Yuan and Yamamoto [6].

Bibliography


Long-time behaviour of the strongly damped semilinear plate equation in $\mathbb{R}^n$

Sema Yayla*
Hacettepe University

We investigate the initial-value problem for the semilinear plate equation containing localized strong damping, localized weak damping and nonlocal nonlinearity. We prove that if nonnegative damping coefficients are strictly positive almost everywhere in the exterior of some ball and the sum of these coefficients is positive a.e. in $\mathbb{R}^n$ then the semigroup generated by the considered problem possesses a global attractor in $H^2(\mathbb{R}^n) \times L^2(\mathbb{R}^n)$. We also establish boundedness of this attractor in $H^3(\mathbb{R}^n) \times H^2(\mathbb{R}^n)$.

Bibliography


We consider a coupled system of PDEs consisting of an incompressible Navier-Stokes equation and a linear damped wave equation coupled through transmission boundary conditions at a moving interface. This is a well-established free moving boundary model describing fluid-elasticity interaction and its local-in-time well-posedness has been treated in several works in the literature \cite{CS1, CS2, KT1, KT2, RV}. In an earlier work, we establish a similar small data global existence result when the system is subject to both boundary and interior damping \cite{IKLT1}. In this work, we establish global-in-time solutions for the system given small initial smooth data and subject only to interior damping but no boundary dissipation, and we show exponential decay of the solution in some Sobolev norm.

We first prove that any smooth solution is global and exponentially decaying if the initial data is small in an appropriate Sobolev norm. The main task is to establish a priori superlinear estimates which guarantee control of the time integral of the wave potential energy. This is achieved using special multipliers and the equipartition energy technique for the wave equation. The second part of the proof concerns the construction of global smooth solutions in a higher norm, to which the first result can be applied in order to conclude exponential decay in the lower Sobolev norm. Here, we appeal to maximal regularity of parabolic equations \cite{PS} especially crafted for Stokes equations with non-homogeneous divergence and Neumann type boundary conditions \cite{MZ1, MZ2}, in addition to sharp trace regularity of solutions to the wave equation \cite{LLT}. This way we are able to construct solutions to the linear system (with given smooth variable coefficients in the Stokes equation). To address the nonlinear system where the variable coefficients in the Lagrangian formulation are also unknown, we use a fixed point iteration scheme to construct global solutions to the full system.

Bibliography


The initial-dynamic boundary value problem (idbvp) for the complex Ginzburg-Landau equation (CGLE) on bounded domains of $\mathbb{R}^N$ is studied by converting the given mathematical model into a Wentzell initial-boundary value problem (ibvp). First, the corresponding linear homogeneous idbvp is considered. Secondly, the forced linear idbvp with both interior and boundary forcings is studied. Then, the nonlinear idbvp with Lipschitz nonlinearity in the interior and monotone nonlinearity on the boundary is analyzed. The local well-posedness of the idbvp for the CGLE with power type nonlinearities is obtained via a contraction mapping argument. Global well-posedness for strong solutions is shown. Global existence and uniqueness of weak solutions are proven. Smoothing effect of the corresponding evolution operator is proved. This helps to get better well-posedness results than the known results on idbvp for nonlinear Schrödinger equations (NLS). An interesting result is proving that solutions of NLS subject to dynamic boundary conditions can be obtained as inviscid limits of the solutions of the CGLE subject to same type of boundary conditions. Finally, long time behaviour of solutions is characterized and exponential decay rates are obtained at the energy level by using control theoretic tools.

Türker Özsarı’s research was supported by TÜBİTAK 3501 Career Grant 115F055
Magnetohydrodynamics (MHD) studies consider the dynamics of electrically conducting fluids. MHD are described by a set of equations, which are a combination of the Navier–Stokes equations of fluid dynamics and Maxwell’s equations of electromagnetism [1]. In most terrestrial applications, MHD flows occur at low magnetic Reynolds numbers. In this study, we apply the finite element method to time-dependent MHD flows with implicit-explicit (IMEX) method to discretization at low magnetic Reynolds number. We introduce an imex (implicit-explicit) method for time discretization and also finite element method for space discretization [2, 3]. Finally, the stability of the present method is presented comprehensively.

**Key words:** Magnetohydrodynamics, Imex methods, Finite Element Method

**Bibliography**


Inverse Source Problem for Benjamin-Bona-Mahony-Burger Equation

Serap Gümüş*
Koç University

We study an inverse source problem for Benjamin-Bona-Mahony-Burger Equation. Our aim is to prove existence, uniqueness of solution, continuous dependence of solutions on initial data and asymptotic behavior of solutions as $t \to \infty$. 
Boosting the decay of solutions of the Korteweg-de Vries-Burgers equation to a predetermined rate from the boundary

Eda Arabacı*, Türker Özsarı
Izmir Institute of Technology

We extend some recent results on the boundary feedback controllability of the Korteweg-de Vries equation (KdV) to the Korteweg-de Vries-Burgers equation (KdVB) which is posed on a bounded domain. In the first part of the talk, we discuss the fact that all sufficiently small solutions can be steered to zero at "any desired" exponential rate by means of a suitably constructed boundary feedback controller. In the second part, an observer is proposed when a type of boundary measurement is available while there is no full access to the medium.
TBA

Varga Kalantarov
Koç University

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