Small Data Global Existence and Exponential Decay of Solutions to a Free Boundary Fluid-Structure Interaction System with Damping[1]

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This is joint work with Igor Kukavica, Mihaela Ignatova and Irena Lasiecka [IKLT2]. We consider a coupled system of PDEs consisting of an incompressible Navier-Stokes equation and a linear damped wave equation coupled through transmission boundary conditions at a moving interface. This is a well-established free moving boundary model describing fluid-elasticity interaction and its local-in-time well-posedness has been treated in several works in the literature [CS1, CS2, KT1, KT2, RV]. In an earlier work, we establish a similar small data global existence result when the system is subject to both boundary and interior damping [IKLT1]. In this work, we establish global-in-time solutions for the system given small initial smooth data and subject only to interior damping but no boundary dissipation, and we show exponential decay of the solution in some Sobolev norm.

We first prove that any smooth solution is global and exponentially decaying if the initial data is small in an appropriate Sobolev norm. The main task is to establish a priori superlinear estimates which guarantee control of the time integral of the wave potential energy. This is achieved using special multipliers and the equipartition energy technique for the wave equation. The second part of the proof concerns the construction of global smooth solutions in a higher norm, to which the first result can be applied in order to conclude exponential decay in the lower Sobolev norm. Here, we appeal to maximal regularity of parabolic equations [PS] especially crafted for Stokes equations with non-homogeneous divergence and Neumann type boundary conditions [MZ1, MZ2], in addition to sharp trace regularity of solutions to the wave equation [LLT]. This way we are able to construct solutions to the linear system (with given smooth variable coefficients in the Stokes equation). To address the nonlinear system where the variable coefficients in the Lagrangian formulation are also unknown, we use a fixed point iteration scheme to construct global solutions to the full system.

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