# NON-HOMOGENEOUS CONTINUOUS AND DISCRETE GRADIENT SYSTEMS: THE QUASI-CONVEX CASE

Vahid Mohebbi[1]

*Instituto Nacional de Matemática Pura e Aplicada (IMPA), Rio de Janeiro, Brazil.*

Abstract. In this paper, first we study the weak and strong convergence of solutions to the following first order nonhomogeneous gradient system

$$\begin{cases} -x'(t) = \nabla\phi(x(t)) + f(t), & \text{a.e. on } (0,\infty) \\ x(0) = x_0 \in H \end{cases}$$

to a critical point of $\phi$, where $\phi$ is a $C^1$ quasi-convex function on a real Hilbert space $H$ with Argmin$\phi \neq \varnothing$. These results extend the results of [**?**] to non-homogeneous case. Then the discrete version of the above system by backward Euler discretization has been studied. Beside of the proof of the existence of the sequence given by the discrete system, some results on the weak and strong convergence to the critical point of $\phi$ are also proved. These results when $\phi$ is pseudo-convex (therefore the critical points are the same minimum points) may be applied in optimization for approximation of a minimum point of $\phi$.

**Keywords:** Gradient system, quasi-convex, backward Euler discretization, weak convergence, strong convergence.

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[1]E-mail: mohebbi@impa.br.