

Existence of Minimal and Maximal solutions for Quasilinear  
Elliptic Equation with Nonlocal Boundary Conditions on  
Time-Scales\*

Mohammed Derhab<sup>1</sup> & Mohammed Nehari<sup>2</sup>

<sup>1</sup>Department of Mathematics

Faculty of Sciences

University Abou-Bekr Belkaid Tlemcen

B.P.119, Tlemcen

13000, Algeria

e-mail: derhab@yahoo.fr

<sup>2</sup>Department of Mathematics

Faculty of Sciences

E.S.M.Tlemcen

Algeria

e-mail: nehari\_72@yahoo.fr

**Abstract.** The purpose of this work is the construction of minimal and maximal solutions for a class of second order quasilinear elliptic equation subject to nonlocal boundary conditions. More specifically, we consider the following nonlinear boundary value problem

$$\begin{cases} -(\varphi_p(u^\Delta))^\Delta = f(x, u), & \text{in } (a, b)_T, \\ u(a) - a_0 u^\Delta(a) = g_0(u), \\ u(\sigma(b)) + a_1 u^\Delta(\sigma(b)) = g_1(u), \end{cases} \quad (1)$$

where  $p > 1$ ,  $\varphi_p(y) = |y|^{p-2}y$ ,  $(\varphi_p(u^\Delta))^\Delta$  is the one-dimensional  $p$ -Laplacian,  $f : [a, b]_T \times \mathbb{R} \rightarrow \mathbb{R}$  is a rd-continuous function,  $g_i : C_{rd}([a, b]_T) \times C_{rd}([a, b]_T) \rightarrow \mathbb{R}$  ( $i = 0$  and  $1$ ) are rd-continuous and  $a_0$  and  $a_1$  are a positive real numbers.

**Key words:** Quasilinear elliptic equation; Time-Scale, Nonlocal boundary conditions; upper and lower solutions; monotone iterative technique

**AMS Classification:** 34B10, 34B15

---

\*Research partially supported by a MESRS-DRS grant B02020100075 and PNR project N<sup>o</sup> 43/28/2011.