Inertial manifolds for reaction-diffusion-advection problems

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An inertial manifold (IM) is one of the key objects in the modern theory of dissipative systems generated by partial differential equations (PDEs) since it allows us to describe limit dynamics of the considered system by the reduced finite-dimensional system of ordinary differential equations (ODEs). It is well known that the existence of an IM is guaranteed when the so called spectral gap conditions are satisfied, whereas their violation leads to the possibility of an infinite dimensional limit dynamics, at least on the level of an abstract parabolic equation. However, these conditions restrict greatly the class of possible applications and are usually satisfied in the case of one spatial dimension only.

The main aim of this talk is a comprehensive study of reaction-diffusion-advection (RDA) equations in 1D with different boundary conditions (BC). Even in 1D case the spectral gap condition does not satisfied for RDA equations and therefore question on existence/non-existence of an IM for it requires accurate analysis. In the case of Dirichlet or Neumann boundary conditions we will show the existence of an IM using a specially designed non-local in space diffeomorphism which transforms the equations to the new ones for which the spectral gap conditions are satisfied. In contrast to this, in the case of periodic boundary conditions, we will construct a natural example of a RDA system which does not possess an IM. It is worth mentioning that scalar RDA equation even with periodic BC possesses IM, and this may be shown by using the modification of the transform proposed for Dirichlet BC.