Error analysis of a semi-discrete method for nonlocal nonlinear wave equations

Husnu Ata ERBAY

In this study we present a semi-discrete numerical method for solving the initial-value problem of the nonlocal nonlinear wave equation [1]

$$u_{tt} = (\beta * f(u))_{xx}.$$

Here the symbol $*$ is used to denote the convolution operation in the spatial domain

$$(\beta * v)(x) = \int_{\mathbb{R}} \beta(x-y)v(y)\mathrm{d}y$$

and the kernel β is an even function with $\int_{\mathbb{R}} \beta(x)dx = 1$. The numerical method is based on a uniform spatial discretization of the convolution integral. We provide a rigorous convergence analysis and demonstrate the second-order accuracy in space. The main ingredient in the convergence analysis is the uniform estimate of the second-order derivative of the kernel function. To confirm our theoretical findings, we apply the scheme to two different situations: the propagation of a single-solitary wave and the finite-time blow-up of solutions.

This is a joint work with Saadet Erbay (Ozyegin University) and Albert Erkip (Sabanci University).

References

[1] N. Duruk, H.A. Erbay and A. Erkip, Global existence and blow-up for a class of nonlocal nonlinear Cauchy problems arising in elasticity, *Nonlinearity* **23** (2010), 107–118.