Analytical solution of the Volterra-Fredholm singular integro-differential

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Abstract

In this paper, we apply Taylor's approximation and then transform the given nth-order weakly singular linear Volterra and Fredholm integro-differential equations with into an ordinary linear differential equation. Some different examples are considered the results of these examples indicated that the procedure of transformation method is simple and effective, and could provide an accurate approximate solution or exact solution.

1 INTRODUCTION

In this paper, we consider the following nth-order linear-weakly singular Fredholm and Volterra integro-differential equations at the form :

$$y^{(n)}(t)+\sum_{m=0}^{n-1}\beta_m(t)y^{(m)}(t)=f(t)+\int_a^b \frac{y^{(p)}(s)}{|t-s|^{\alpha}}ds, \quad a\le t\le b, \quad 0<\alpha<1 \quad (1)$$

with initial conditions

$$y(a)=\mu_0, y'(a)=\mu_1, ..., y^{(n-1)}(a)=\mu_{n-1} \quad (2)$$

$$y^{(n)}(t)+\sum_{m=0}^{n-1}\beta_m(t)y^{(m)}(t)=f(t)+\int_a^t \frac{y^{(p)}(s)}{(t-s)^{\alpha}}ds, \quad a\le t\le b, \quad 0<\alpha<1 \quad (3)$$

with initial conditions

$$y(a)=\mu_0, y'(a)=\mu_1, ..., y^{(n-1)}(a)=\mu_{n-1} \quad (4)$$

where $\mu_i(i=1,....,n-1)$; are real constants, n; p are positive integers and $0\le p\le n$, $f(t)$; $\beta_k(t)(k=0,1,...,n-1)$ are given functions, and $y(t)$ is the solution to be determined.

The objectives of this work is to develop a new approach to resolve the high-order linear weakly-singular Fredholm and Volterra integro-differential equations in one dimensional space.The proposed method is a direct method in which we remove the singularity by using Taylor's approximation and rewrite the weakly-singular integro-differential Fredholm and Volterra equation as either a linear differential equation that can be solved using the analytical method or the known numerical methods.

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